Online Appendix to “Competing under Information Heterogeneity: Evidence from Auto Insurance”*

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The opinions and errors in this paper are solely those of the authors and do not reflect the views of IVASS and the Bank of Italy.

A Additional Tables

Table A.1: Selected variables used by five major auto insurance companies

Firm A	**Firm B**	**Firm C**	**Firm D**	**Firm E**
Years of non-circulation of the vehicle	Safe Driving & Savings Clause	Brand-model of the vehicle	Credit information of the vehicle owner (data derived from the census cell of residence)	Census cells
Vehicle weight	License Seniority / Type of Person / CU Class	Age of the vehicle at the time of purchase	Use	Special uses (Driving school, Leasing, etc.)
Brand-model of the vehicle	Infocar Code	Occupation	Trailer towing	Vehicle adapted for reduced motor capacity
Years of vehicle ownership	Vehicle Age at Purchase	Marital status	Family parameter (presence of other car insurance policies within the family unit)	Number of driving wheels
Purchase of a new/used vehicle	Increase for vehicles already insured with NA in the Current Year	Seniority in obtaining a driving license	Safe driving	Body type
Occupation	Waiver of Right to Recourse Clauses	Kilometers per year	Discount program for youth	Safety devices
			Temporary insurance	Brand
			Vehicle size category	Transported substances

Note: This table reports a subset of the pricing variables used by five major auto insurance companies. To protect firm confidentiality and ensure anonymity, only a selected subset of variables is presented. These variables are not observed in our dataset. The types of information collected from consumers by insurers are generally accessible through their online quoting systems available on company websites.

Table A.2: Regression of premiums on observable characteristics

	Premium	Premium	Premium	Premium	Premium	Premium
VARIABLES	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	Firm 6
Age	-1.0561***	-0.7269***	-1.5116***	-0.2459	-0.4106**	-0.7838***
	(0.0888)	(0.1510)	(0.1312)	(0.1609)	(0.1734)	(0.0744)
BM	19.7425***	24.9416***	26.4326***	20.9453***	31.0295***	21.8787***
	(0.8648)	(1.1237)	(1.1970)	(1.9128)	(1.5464)	(0.4911)
Man	-8.9443***	-7.8892*	-2.6870	-14.7678***	-6.2815	3.9914**
	(2.3646)	(4.0410)	(3.6989)	(4.6169)	(4.9169)	(1.8632)
1 Acc.	124.1271***	149.2277***	106.1311***	74.1313***	112.2941***	111.7719***
	(4.0282)	(7.0702)	(5.6987)	(6.3382)	(8.8072)	(3.0169)
Big city	34.4064***	45.6070***	56.5082***	55.2540***	39.8449***	16.9238***
	(2.5688)	(4.3682)	(3.8889)	(4.4827)	(5.0987)	(2.0101)
Constant	639.0548***	620.9940***	575.3849***	515.6197***	594.6249***	511.3076***
	(16.5555)	(35.6947)	(26.2550)	(59.5533)	(86.1046)	(20.6469)
R-squared	0.3974	0.3903	0.4750	0.4644	0.4763	0.5887

VARIABLES	Firm 7	Firm 8	Firm 9	Firm 10	Firm 11	All Firms
Age	-1.3800***	-1.6772***	-0.9907***	-1.0137***	-0.5548***	-0.7635***
	(0.2277)	(0.2072)	(0.3104)	(0.2076)	(0.0468)	(0.0330)
BM	23.6438***	12.9392***	31.5349***	12.0464***	23.1305***	21.9900***
	(1.5264)	(2.1503)	(2.3238)	(2.0040)	(0.4227)	(0.2984)
Man	6.8855	-1.2936	-9.7677	-9.4804	-0.6410	-4.8530***
	(6.5698)	(5.6599)	(8.5327)	(6.1557)	(1.2818)	(0.8964)
1 Acc.	125.2567***	118.8831***	147.2334***	181.3849***	119.4853***	121.7660***
	(9.9382)	(9.1527)	(16.4997)	(12.3331)	(2.0065)	(1.4266)
Big city	48.7960***	56.1094***	84.5230***	52.8687***	12.3459***	28.6369***
	(7.3501)	(6.2824)	(10.1850)	(6.7416)	(1.4467)	(0.9829)
Constant	499.9007***	697.3515***	157.8634**	694.9461***	449.3726***	516.3972***
	(34.4620)	(26.2524)	(75.7489)	(60.5639)	(8.6075)	(7.1540)
R-squared	0.4515	0.3992	0.5101	0.4458	0.5234	0.4834

Note: All regressions reported in this table control for vehicle characteristics, time fixed effects, and optional contract clauses. The number of observations in each regression is omitted to prevent the identity of the firm from being inferred through market shares.

Table A.3: Summary statistics

Variables	Mean	Std. Dev.	Min	Max	N
Premium (€)	477.68	208.79	133.68	1335.05	124,428
Claim size (€)	260.89	10217.58	0	2521014	124,428
Number of claims (within contract year)	0.08	0.29	0	4	124,428
Number of accidents in last 5 years	0.81	1.22	0	3	124,428
BM class	2.06	2.51	1	15	124,428
Age	48.24	14.11	18	99	124,428
Man	0.56	0.50	0	1	124,428
Median urban center	0.10	0.30	0	1	124,428
Larger urban center	0.62	0.49	0	1	124,428
Car age	8.30	5.27	0	19	124,428
Horsepower	66.88	26.84	0	493	124,428
Petrol vehicle	0.52	0.50	0	1	124,428
One installment	0.67	0.47	0	1	124,428

Table A.4: Regression of premiums on estimated risk type and observable characteristics

	Premium	Premium	Premium	Premium	Premium	Premium
VARIABLES	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	Firm 6
Est. Risk	0.0642***	0.0936***	0.0572***	0.0430***	0.0515***	0.0731***
	(0.0042)	(0.0068)	(0.0064)	(0.0073)	(0.0100)	(0.0028)
Age	-0.8955***	-0.4840***	-1.3915***	-0.1352	-0.3352*	-0.5922***
	(0.0889)	(0.1500)	(0.1315)	(0.1611)	(0.1737)	(0.0731)
BM	19.1538***	24.5213***	26.0305***	20.6462***	30.7392***	21.6734***
	(0.8671)	(1.1160)	(1.1907)	(1.9027)	(1.5478)	(0.4715)
Man	-10.7472***	-7.3841*	-3.9804	-15.4388***	-7.0770	2.9258
	(2.3454)	(3.9867)	(3.6796)	(4.6002)	(4.8897)	(1.8135)
1 Acc.	113.0035***	129.5094***	94.8281***	66.1749***	102.6605***	99.7637***
	(4.0816)	(7.1153)	(5.8039)	(6.3675)	(8.8333)	(2.9952)
Big city	34.3785***	46.6072***	56.8048***	55.6861***	39.8474***	16.7550***
	(2.5445)	(4.3043)	(3.8697)	(4.4581)	(5.0768)	(1.9533)
Constant	632.2263***	619.6198***	571.1734***	514.9453***	607.4646***	496.8311***
	(16.4609)	(35.4253)	(26.3254)	(60.2578)	(85.1550)	(18.5252)
R-squared	0.4057	0.4077	0.4814	0.4690	0.4814	0.6117
VARIABLES	Firm 7	Firm 8	Firm 9	Firm 10	Firm 11	All Firms
Est. Risk	0.0767***	0.0459***	0.0784***	0.0387***	0.0823***	0.0731***
	(0.0111)	(0.0090)	(0.0173)	(0.0118)	(0.0023)	(0.0015)
Age	-1.1958***	-1.5677***	-0.9027***	-0.9363***	-0.3337***	-0.5777***
	(0.2298)	(0.2083)	(0.3089)	(0.2083)	(0.0464)	(0.0329)
BM	23.4924***	12.8527***	31.4627***	11.6583***	22.2223***	21.3837***
	(1.5431)	(2.1425)	(2.3038)	(2.0099)	(0.4208)	(0.2974)
Man	6.1553	-1.9849	-9.5761	-9.4334	-2.0086	-6.0691***
	(6.5007)	(5.6374)	(8.4552)	(6.1263)	(1.2572)	(0.8847)
1 Acc	113.1750***	110.7144***	131.3323***	173.2216***	106.8438***	109.3707***
	(10.0975)	(9.2865)	(16.4076)	(12.6389)	(1.9965)	(1.4293)
Big city	48.1035***	56.8488***	85.4458***	53.4525***	12.9113***	28.9435***
	(7.2178)	(6.2390)	(10.1483)	(6.7341)	(1.4208)	(0.9704)
Constant	494.5030***	697.9200***	167.8205**	692.1357***	443.5997***	511.0770***
	(33.9842)	(26.2245)	(75.6721)	(60.4907)	(8.4378)	(7.0944)
R-squared	0.4640	0.4037	0.5210	0.4489	0.5403	0.4957

Note: All regressions reported in this table control for vehicle characteristics, time fixed effects, and optional contract clauses. The number of observations in each regression is omitted to prevent the identity of the firm from being inferred through market shares.

Table A.5: Poisson regression of claim count on premium and observable characteristics

	Claim Count	Claim Count	Claim Count	Claim Count	Claim Count	Claim Count
VARIABLES	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	Firm 6
Premium	0.8936***	1.1735***	0.4965**	1.6590***	0.7366*	0.5847***
	(0.1401)	(0.2152)	(0.2375)	(0.2927)	(0.3768)	(0.2105)
Age	-0.0047**	-0.0027	-0.0018	-0.0104**	-0.0105**	-0.0051**
	(0.0019)	(0.0031)	(0.0028)	(0.0042)	(0.0043)	(0.0022)
BM	-0.0093	-0.0102	0.0243	-0.0330	-0.0005	-0.0040
	(0.0108)	(0.0180)	(0.0164)	(0.0226)	(0.0287)	(0.0112)
Man	-0.0340	-0.1903**	-0.0257	-0.1226	0.2131*	-0.1083*
	(0.0529)	(0.0892)	(0.0816)	(0.1134)	(0.1247)	(0.0574)
1 Acc.	0.2601***	0.2624**	0.1289	0.1168	0.3383*	0.2681***
	(0.0821)	(0.1260)	(0.1208)	(0.1586)	(0.1819)	(0.0805)
Big city	0.0671	0.0450	0.2042**	-0.0558	0.2835**	0.1360**
	(0.0594)	(0.0960)	(0.0924)	(0.1147)	(0.1328)	(0.0640)
Constant	-2.1891***	-3.3079***	-1.9759***	-2.7247***	-2.9402***	-1.8888***
	(0.2750)	(1.1186)	(0.5409)	(0.4606)	(0.5679)	(0.6591)
VARIABLES	Firm 7	Firm 8	Firm 9	Firm 10	Firm 11	All Firms
Premium	1.1908***	0.8987***	1.8645***	1.0451***	0.8972***	0.8753***
	(0.4378)	(0.2864)	(0.5203)	(0.3929)	(0.1042)	(0.0611)
Age	-0.0015	-0.0000	0.0122*	0.0054	-0.0056***	-0.0046***
	(0.0049)	(0.0038)	(0.0066)	(0.0048)	(0.0013)	(0.0008)
BM	0.0210	-0.0502**	-0.0507	-0.0276	0.0036	-0.0024
	(0.0285)	(0.0252)	(0.0401)	(0.0287)	(0.0066)	(0.0042)
Man	-0.1769	-0.1955*	0.0228	-0.0971	-0.0825**	-0.0784***
	(0.1448)	(0.1046)	(0.1888)	(0.1423)	(0.0345)	(0.0216)
1 Acc.	-0.2171	0.1641	-0.0928	0.0366	0.1480***	0.1861***
	(0.2198)	(0.1702)	(0.3303)	(0.2549)	(0.0494)	(0.0315)
Big city	-0.0218	0.0968	-0.3329	0.0082	0.1000**	0.0977***
	(0.1691)	(0.1217)	(0.2055)	(0.1605)	(0.0403)	(0.0243)
Constant	-2.6005***	-2.1402***	-2.9924***	-2.8259**	-2.5547***	-2.7121***
	(0.6043)	(0.4632)	(0.5778)	(1.1680)	(0.2066)	(0.1296)

Note: All regressions reported in this table control for vehicle characteristics, time fixed effects, and optional contract clauses. Premiums are represented in thousands of euros. The number of observations in each regression is omitted to prevent the identity of the firm from being inferred through market shares.

Table A.6: Correlation coefficients between firms' signal standard deviations, marginal costs, and claim processing efficiency.

	σ_j	mc_j	k_j
σ_j	1.00		
mc_j	-0.72	1.00	
	(0.01)		
k_j	0.64	-0.75	1.00
	(0.03)	(0.01)	

Table A.7: Counterfactual results: Distributional effects on consumer surplus

	Baseline	Observing True Risk	Centralized Risk Bureau	Privacy Regulation
Average CS: Young drivers (€)	-732.31	-609.75	-603.37	-706.81
		(+16.74%)	(+17.61%)	(+3.48%)
Average CS: Senior drivers (€)	-366.90	-303.70	-322.24	-353.16
		(+17.23%)	(+12.17%)	(+3.75%)
Average CS: Smaller urban centers (€)	-539.12	-483.16	-476.27	-525.56
		(+10.38%)	(+11.66%)	(+2.52%)
Average CS: Larger urban centers (€)	-545.10	-431.35	-446.14	-522.11
		(+20.87%)	(+18.16%)	(+4.22%)

Table A.8: Percentage changes in counterfactual market outcomes: Comparison of information-alone and full-equilibrium outcomes under the centralized bureau policy.

	Improved Information Alone	Centralized Risk Bureau
Average CS	2.78	15.70
Average CS: Low risk	47.89	78.28
Average CS: High risk	-32.59	-33.39
Average premium	-11.57	-21.60
Average profit	-2.14	-5.92
HHI	-8.41	-1.44
Average cost	-0.55	-1.41

Note: In the information-alone scenario, all firms receive the aggregated risk evaluations from the centralized bureau but maintain their existing pricing strategies.

B Estimating Individual-Specific Risk

In this section, we describe how we estimate an individual-specific risk measure using a panel dataset of claim records, which includes the number of accidents and the amount paid to consumers following an accident in each contract year. The estimated individual-level risk is used in the reduced-form analysis presented in Section 2.2 of the main text.

Let i be the index of each customer and t be the index of a contract year. For each consumer in the dataset, we observe a vector of characteristics (such as age, vehicle features, geographic locations, and contractual clauses), which we denote by $\boldsymbol{X}_{it}$. We first estimate a Poisson regression of claim counts on observables with an individual fixed effect. Specifically,

$$E[ClaimCount_{it}|\boldsymbol{X}_{it}, \psi_i] = \exp(\boldsymbol{X}_{it}\delta_c + \psi_i). \tag{B.1}$$

We then estimate a log-normal regression of claim size conditional on the consumer being involved in an accident:[1]

$$\log(ClaimSize_{it}) = \delta_0 + \boldsymbol{X}_{it}\delta_s + \eta_{it}. \tag{B.2}$$

With the estimated regression coefficients and the individual fixed effects from Equations (B.1) and (B.2), we predict the expected number of accidents and the expected indemnity conditional on involvement in an accident for each consumer. Multiplying the two yields an estimate for the *expected* cost of insuring consumer i in a year.[2] Since we control for contract features in these regressions, the impact of consumers self-selecting into different clauses has been factored into our risk estimates.[3]

[1]We assume that claim size is independent of the individual fixed effect following Jeziorski et al. (2017). This assumption is motivated by the arguments in the actuarial literature that accident severity is more random and less related to the individual's driving ability.

[2]Our approach to estimating the expected cost of insuring a consumer is related to the two methods discussed in Abaluck and Gruber (2016). The first is a "realized cost" model, which constructs out-of-pocket costs using claims incurred during the year. The second is a "rational expectations" model, which predicts expected drug spending based on claims from the prior year. Our approach can be viewed as a hybrid of these two methods. We estimate the expected cost of insuring a consumer using ex-post realized claim records, allowing the estimation to depend not only on rich consumer characteristics, including accident histories, but also on individual fixed effects. The key advantage of our setting is that we observe each consumer over multiple years, which enables us to control for individual fixed effects and better capture heterogeneity that may not be fully explained by observables alone.

[3]Specifically, we control for contract features including coverage, repair restrictions, exclusive driving, expert driving, free driving, protected bonus, and the presence of a monitoring device. Our estimates of risk can be interpreted as incorporating the effects of moral hazard in a reduced-form way. That is, consumers may change their risky behavior after choosing different contracts.

C Estimation Details

We describe the step-by-step estimation procedure for the model. Let $i = 1, 2, \ldots, N$ index individual consumers. For each consumer i, we observe the number of accidents in period t, denoted by y_{it} for $t = 1, 2, \ldots, T$. Let p_i denote the premium paid, and $D_i \in \{1, 2, \ldots, J\}$ represent the contract choice of consumer i. The estimation procedure described in this section can be carried out conditional on observable consumer characteristics, which we omit for notational simplicity.

Step 1: Recovering Risk Type Distribution We assume that the Poisson rate can take on a finite set of values $\{\lambda_1, \lambda_2, \ldots, \lambda_L\}$. Let $\boldsymbol{q} = (q_1, q_2, \ldots, q_L)$ denote the vector of probabilities associated with each Poisson rate such that $\sum_{l=1}^{L} q_l = 1$ and $q_l \geq 0, \forall l$. Define $I_j(\underline{p}, \bar{p})$ as the set of consumer indices such that for all $i \in I_j(\underline{p}, \bar{p})$, we have $p_i \in [\underline{p}, \bar{p}]$ and $D_i = j$. We estimate the probabilities of each Poisson rate for this group of consumers by maximizing the following log-likelihood function.[4]

$$LL_{\lambda}(\boldsymbol{q}; I_j(\underline{p}, \bar{p})) = \sum_{i \in I_j(\underline{p}, \bar{p})} \log \left(\sum_{l=1}^{L} \frac{\lambda_l^{(\sum_{t=1}^{T} y_{it})} \exp(-\lambda_l T)}{\prod_{t=1}^{T} y_{it}!} q_l \right).$$

Given the estimated probability distribution of the Poisson rate for consumers within each group, we randomly draw a Poisson rate $\tilde{\lambda}_i$ for each consumer i. We then construct a simulated risk type for that consumer as $\tilde{\theta}_i = \hat{\mu}\tilde{\lambda}_i$, where $\hat{\mu}$ is the estimated expected claim size based on the log-normal regression in Equation (B.2). Using the sample $(p_i, \tilde{\theta}_i, D_i)$, we estimate the density of premiums conditional on risk type and contract choice. Specifically, we discretize risk types into 20 bins and apply kernel density estimation to obtain $\hat{g}(p|\theta, D = j)$, the estimated premium density conditional on each risk type and the consumer's chosen contract. This serves as the output of the first-step estimation.

This simulation-based approach to estimating $g(p|\theta, D = j)$ introduces noise due to the random draws of $\tilde{\lambda}_i$. Alternatively, once we have recovered the distribution of $\lambda|p, D = j$, together with the distribution of μ, we can derive the distribution of $\theta|p, D = j$. Then, $g(p|\theta, D = j)$ can be obtained directly using Bayes' rule, with the marginal distribution of p estimated nonparametrically from the data. We confirm that this alternative method produces results for $g(p|\theta, D = j)$ similar to our approach. However, because we use

[4] In practice, we discretize the observed premiums into 100 bins, each with a width of approximately 23 euros. We experiment with allowing the Poisson rate to take between 20 and 60 discrete values and estimate the model and conduct the counterfactual analysis under each specification. The results are robust: both the parameter estimates and the key counterfactual implications remain stable. In the main text, we report the results based on 50 discrete values for the Poisson rate.

a kernel estimate of $g(p|\theta, D = j)$ based on the simulated dataset, our approach yields a smoother density function by construction. In addition, assigning a simulated draw for each individual simplifies the maximum likelihood estimation of the demand- and supply-side parameters parameters. For these reasons, we report the main estimation results using the simulation approach.

Step 2: Estimating Demand Parameters We use a nested fixed-point algorithm to estimate the demand parameters, taking the first-step estimates $\hat{g}(p|\theta, D = j)$ as an input. In the inner loop, we fix the price sensitivity parameter γ and apply the iterative procedure described in Section 4.2 to solve for the vector of unobserved product attributes $\boldsymbol{\xi}(\gamma)$ and the offered price distribution $g_j(p|\theta;\gamma)$. We evaluate the offered price distribution at 2,500 grid points. Convergence of the fixed-point algorithm requires that, for all firms and at all grid points, the difference between successive iterations falls below a pre-specified tolerance level.

In the outer loop, we estimate γ by maximizing the following log-likelihood function.

$$LL_d(\gamma) = \sum_{i=1}^{N}\sum_{j=1}^{J} \mathbf{1}\{D_i = j\} \log\left(Pr(D_i = j|\tilde{\theta}_i;\gamma)\right),$$

where $Pr(D_i = j|\tilde{\theta}_i;\gamma)$ denotes the model-implied probability that consumer i chooses a contract from firm j conditional on risk type $\tilde{\theta}_i$, and is constructed as follows:

$$Pr(D = j|\theta;\gamma) = \int_{\boldsymbol{p}} \frac{\exp(-\gamma p_j + \xi_j(\gamma))}{\sum_{j'} \exp(-\gamma p_{j'} + \xi_{j'}(\gamma))} \left(\prod_{j'} g_{j'}(p_{j'}|\theta;\gamma)\right) d\boldsymbol{p}.$$

To compute the choice probability $Pr(D = j|\theta;\gamma)$, we simulate offered prices from $g_j(p|\theta;\gamma)$ and evaluate the probabilities using numerical integration. Once we obtain an estimate $\hat{\gamma}$ for the price sensitivity parameter, the vector of unobserved product attributes $\boldsymbol{\xi}(\hat{\gamma})$ and the offered price distribution $g_j(p|\theta;\hat{\gamma})$ are recovered as by-products.

Step 3: Estimating Pricing Coefficients Equations (4.4) and (4.5) uniquely determine α_j and β_j as follows:

$$\alpha_j = E(p|D = j) - \frac{var(p|D = j)}{cov(p, \theta|D = j)} E(\theta|D = j),$$
$$\beta_j = \frac{var(p|D = j)}{cov(p, \theta|D = j)}.$$

Using the sample $(p_i, \tilde{\theta}_i, D_i)$, we estimate $E(p|D=j)$, $E(\theta|D=j)$, $var(p|D=j)$ and $cov(p,\theta|D=j)$ and use these estimates to compute $\hat{\alpha}_j$ and $\hat{\beta}_j$.

Alternatively, α_j and β_j can be identified and estimated using a linear regression of θ on prices p_j. To see this, note that the pricing equation (3.2) is equivalent to the following:

$$\theta = -\frac{\alpha_j}{\beta_j} + \frac{p_j}{\beta_j} + \underbrace{(\theta - E(\theta|\hat{\theta}_j, D=j))}_{\text{error term}},$$

where $cov(p_j, \theta - E(\theta|\hat{\theta}_j, D=j)|D=j) = 0$. Regressing θ on p_j for consumers within firm j identifies the coefficients $-\frac{\alpha_j}{\beta_j}$ and $\frac{1}{\beta_j}$.

Step 4: Estimating Signal Variance We first derive the posterior joint distribution of signals and risk types for any given σ_j using the Bayes rule:

$$f(\hat{\theta}_j, \theta|D=j;\sigma_j) = \frac{Pr(D=j|p_j^o(\hat{\theta}_j;\theta,\sigma_j),\theta)\phi(\hat{\theta}_j;\theta,\sigma_j)f_0(\theta)}{s_j},$$

where $Pr(D=j|p_j^o(\hat{\theta}_j;\theta,\sigma_j),\theta)$ represents the likelihood of selecting firm j conditional on the signal $\hat{\theta}_j$ (or equivalently the corresponding price $p_j^o(\hat{\theta}_j;\theta,\sigma_j)$) and type θ. The denominator s_j represents the market share of firm j and serves as a normalization factor.

Next, we derive the model-generated price as a function of the signal, which we denote by $gp_j(\hat{\theta}_j;\sigma_j)$, taking the estimated pricing coefficients $\hat{\alpha}_j$ and $\hat{\beta}_j$ as given. Specifically,

$$gp_j(\hat{\theta}_j;\sigma_j) = \hat{\alpha}_j + \hat{\beta}_j E(\theta|\hat{\theta}_j, D=j;\sigma_j), \tag{C.1}$$

where $E(\theta|\hat{\theta}_j, D=j;\sigma_j)$ represents the equilibrium risk rating given signal $\hat{\theta}_j$ as a function of σ_j (see Equation (4.8)).

Let gp_j^{-1} denote the inverse function of gp_j in Equation (C.1) to back out the signal corresponding to an observed premium. Then using the change of variables formula, we construct the model-implied posterior joint distribution of the premiums and risk types conditional on the consumers being selected into firm j:

$$h(p,\theta|D=j;\sigma_j) = \frac{f(gp_j^{-1}(p;\sigma_j),\theta|D=j;\sigma_j)}{gp'(gp_j^{-1}(p;\sigma_j);\sigma_j)}.$$

We estimate σ_j for each firm j by maximizing the following likelihood function:

$$LL_p(\sigma_j) = \sum_{i=1}^{N} \mathbf{1}\{D_i=j\} \log\Big(h(p_i,\tilde{\theta}_i|D=j;\sigma_j)\Big).$$

Step 5: Estimating Cost Parameters To simplify notation, we use $f(\hat{\boldsymbol{\theta}}|\theta)$ to represent the joint density of signals conditional on the type θ, i.e., $f(\hat{\boldsymbol{\theta}}|\theta) = \prod_{j'=1}^{J} \phi(\hat{\theta}_{j'}; \theta, \sigma_{j'})$. Taking the first-order derivatives of the profit function in Equation (3.5) with respect to (α_j, β_j) yields:

$$
\begin{aligned}
\frac{\partial \pi_j(\boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial \alpha_j} &= \int_\theta \int_{\hat{\boldsymbol{\theta}}} Pr(D = j|\hat{\boldsymbol{\theta}}, \theta) f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta \\
&+ \quad \int_\theta \int_{\hat{\boldsymbol{\theta}}} (\alpha_j + \beta_j \theta + c_j - k_j \theta) \frac{\partial Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \alpha_j} f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta, \quad \text{(C.2)} \\
\frac{\partial \pi_j(\boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial \beta_j} &= \int_\theta \int_{\hat{\boldsymbol{\theta}}} \theta Pr(D = j|\hat{\boldsymbol{\theta}}, \theta) f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta \\
&+ \quad \int_\theta \int_{\hat{\boldsymbol{\theta}}} (\alpha_j + \beta_j \theta + c_j - k_j \theta) \frac{\partial Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \beta_j} f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta. \quad \text{(C.3)}
\end{aligned}
$$

In Equations (C.2) and (C.3), the first terms capture the direct effects of changing the pricing coefficients on profit, while the second terms quantify the marginal changes to the profit through indirect sorting effects.

The key challenge in evaluating the first-order conditions lies in estimating the derivatives of the sorting probabilities with respect to α_j and β_j. Given the equilibrium concept we use (i.e., Nash equilibrium), changing firm j's pricing coefficients does not affect other firms' ($j' \neq j$) pricing strategy $p_{j'}(\hat{\theta}_{j'})$. In other words, other firms will keep using the same pricing strategies they currently use. However, changing firm j's pricing coefficients affects its own prices through two channels: (1) the direct effect, and (2) the indirect effect through the equilibrium risk rating $E(\theta|\hat{\theta}_j, D = j)$. To see this,

$$
\begin{aligned}
\frac{\partial Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \alpha_j} &= -\gamma(\theta) \frac{\partial p_j(\hat{\theta}_j)}{\partial \alpha_j} Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)(1 - Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)) \\
&= -\gamma(\theta) \left[1 + \beta_j \frac{\partial E(\theta|\hat{\theta}_j, D = j)}{\partial \alpha_j} \right] Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)(1 - Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)) \\
&\approx -\gamma(\theta) Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)(1 - Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)). \qquad \text{(C.4)}
\end{aligned}
$$

The approximation in Equation (C.4) assumes that the impact of α_j on its own expected risk level is small. Similarly, we approximate the derivatives of sorting probabilities with respect to β_j in the following equation:

$$
\frac{\partial Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \beta_j} \approx -\gamma(\theta) E(\theta|\hat{\theta}_j, D = j) Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)(1 - Pr(D = j|\hat{\boldsymbol{\theta}}, \theta)). \quad \text{(C.5)}
$$

We verify the approximations in these two equations numerically by varying α_j and β_j for a single firm while holding all other firms' pricing strategies fixed. We then iterate to compute the new equilibrium risk rating $E(\theta|\hat{\theta}_j, D = j)$ using Step 2 of the iterative procedure described in Appendix F. We find that a 1% increase in α_j and β_j changes $E(\theta|\hat{\theta}_j, D = j)$ by an average of 0.01% and 0.04%, respectively. Our results confirm that the effect of α_j and β_j on the firm's own expected equilibrium risk rating is indeed small and is dominated by the direct effect of changing these pricing coefficients.

With the approximations in Equations (C.4) and (C.5), the first-order conditions in Equation (C.2) and (C.3) are reduced to a system of two linear equations involving (c_j, k_j). These two equations uniquely pin down (c_j, k_j) as the solution to the system of linear equations, where all other terms are either directly estimable from the data or have been recovered in previous steps.

Recovering Marginal Cost Let I_j denote the set of consumers who choose firm j in our sample. For each of these consumers, we observe the premium they pay, the actual claim costs, and whether they stay in firm j over the next T years. We can therefore compute the sum of the discounted future premiums averaged across all consumers in I_j, which we denote by $\overline{P}_j$. Specifically,

$$\overline{P}_j = \frac{\sum_{i \in I_j} \sum_t^T \delta^t p_{it} \mathbf{1}\{i \text{ stays in firm } j \text{ at } t\}}{|I_j|}, \tag{C.6}$$

where the discount factor δ is set to 0.95. We use a similar formula as in Equation (C.6) to compute the sum of discounted claim costs and contract sales, which we denote by $\overline{CS}_j$ and $\overline{N}_j$, respectively. Let mc_j denote firm j's marginal cost of managing a contract. The following equation computes the net benefit firms receive from contracting with a new customer:

$$\underbrace{(\overline{P}_j - \overline{CS}_j k_j - \overline{N}_j mc_j)}_{\text{discounted value of future profits}} - mc_j = c_j. \tag{C.7}$$

Equation (C.7) isolates the firm's marginal cost mc_j partially from the dynamic factors. In practice, if insurance companies offer additional products to consumers, they may gain extra benefits from bundling or cross-selling, making our marginal cost estimates a lower bound.

Computing Standard Errors We compute standard errors for the demand- and supply-side parameter estimates using 200 bootstrap replications. In each replication, we resample individuals with replacement from the original dataset, preserving all observations associated with each selected individual. For each bootstrap sample, we repeat the full estimation procedure (Steps 1–5 described above) to recover the model primitives. We then compute the standard deviation of the resulting parameter estimates across replications to obtain the bootstrap standard errors.

D Identification of Signal Variance

We consider the identification of the variance of the signal distribution. The pricing coefficients have been recovered in the previous step and are thus treated as known. For simplicity, we consider the case where the demand parameters do not vary with risk type.

Under the assumption of normally distributed signals, the density function of firm j's signal distribution takes the following form:

$$\phi(\hat{\theta}_j;\theta,\sigma_j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(\hat{\theta}_j - \theta)^2}{2\sigma_j^2} \right).$$

We first derive the posterior density of $\hat{\theta}_j$ for those who self select into firm j. Let $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_J)$ denote the vector of signals received by all firms. Let $\hat{\boldsymbol{\theta}}_{-j}$ denote the vector of signals excluding firm j's signal. To simplify notation, we use $f(\hat{\boldsymbol{\theta}}|\theta)$ and $f(\hat{\boldsymbol{\theta}}_{-j}|\theta)$, respectively, to denote the densities of $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\theta}}_{-j}$ conditional on θ.

$$\begin{aligned} f(\hat{\theta}_j|\theta, D=j) &= \frac{\int_{\hat{\boldsymbol{\theta}}_{-j}} Pr(D=j|\hat{\boldsymbol{\theta}}) f(\hat{\boldsymbol{\theta}}|\theta) d\hat{\boldsymbol{\theta}}_{-j}}{\int_{\hat{\boldsymbol{\theta}}} Pr(D=j|\hat{\boldsymbol{\theta}}) f(\hat{\boldsymbol{\theta}}|\theta) d\hat{\boldsymbol{\theta}}} \\ &= \frac{\exp(-\gamma p_j(\hat{\theta}_j) + \xi_j)\phi(\hat{\theta}_j;\theta,\sigma_j)\left[\int_{\hat{\boldsymbol{\theta}}_{-j}} \frac{f(\hat{\boldsymbol{\theta}}_{-j}|\theta)}{\sum_{j'=1}^{J}\exp(-\gamma p_{j'}(\hat{\theta}_{j'})+\xi_{j'})} d\hat{\boldsymbol{\theta}}_{-j}\right]}{\int_{\hat{\theta}_j} \exp(-\gamma p_j(\hat{\theta}_j) + \xi_j)\phi(\hat{\theta}_j;\theta,\sigma_j)\left[\int_{\hat{\boldsymbol{\theta}}_{-j}} \frac{f(\hat{\boldsymbol{\theta}}_{-j}|\theta)}{\sum_{j'=1}^{J}\exp(-\gamma p_{j'}(\hat{\theta}_{j'})+\xi_{j'})} d\hat{\boldsymbol{\theta}}_{-j}\right] d\hat{\theta}_j} \quad \text{(D.1)} \\ &\approx \frac{\exp(-\gamma p_j(\hat{\theta}_j))\phi(\hat{\theta}_j;\theta,\sigma_j)}{\int_{\hat{\theta}_j} \exp(-\gamma p_j(\hat{\theta}_j))\phi(\hat{\theta}_j;\theta,\sigma_j) d\hat{\theta}_j}. \quad \text{(D.2)} \end{aligned}$$

Denote the term in square brackets in Equation (D.1) by $\Delta(\hat{\theta}_j, \theta)$. Since it is inside of the integral in the denominator, $\Delta(\hat{\theta}_j, \theta)$ cannot be cancelled out. However, if firm j has a small market share, the effect of $\hat{\theta}_j$ on $\Delta(\hat{\theta}_j, \theta)$ is small, and therefore we can treat this term as if it does not depend on $\hat{\theta}_j$. With this approximation, we obtain Equation (D.2).

If the risk-rating term $E(\theta|\hat{\theta}_j, D=j)$ can be well approximated by a linear function of the signal $\hat{\theta}_j$,[5] i.e., $E(\theta|\hat{\theta}_j, D=j) \approx a_j + b_j\hat{\theta}_j$, we can further simplify the posterior

[5] We empirically verify this assumption in Figure E.1, where we plot the relationship between risk rating $E(\theta|\hat{\theta}_j, D=j)$ and signal $\hat{\theta}_j$ after we estimate the model. This figure suggests that the assumption that risk rating can be approximated by a linear function of the signal is reasonable.

distribution of $\hat{\theta}_j$ to be

$$f(\hat{\theta}_j|\theta, D=j) \approx \frac{\exp(-\gamma\beta_j b_j \hat{\theta}_j)\phi(\hat{\theta}_j;\theta,\sigma_j)}{\int_{\hat{\theta}_j}\exp(-\gamma\beta_j b_j \hat{\theta}_j)\phi(\hat{\theta}_j;\theta,\sigma_j)d\hat{\theta}_j}$$
$$= \frac{1}{\sqrt{2\pi\sigma_j^2}}\exp\left(-\frac{(\hat{\theta}_j-(\theta-\gamma\beta_j b_j\sigma_j^2))^2}{2\sigma_j^2}\right). \quad \text{(D.3)}$$

Equation (D.3) implies that after selection, the signal still approximately follows a normal distribution with the same variance (σ_j^2) as before. However, the mean of the posterior normal distribution shifts to $\theta - \gamma\beta_j b_j \sigma_j^2$, which is lower than the original mean θ. This is intuitive as consumers who receive lower prices from j are more likely to self-select into the firm.

The following lemma provides the key identification argument for the signal variance. We focus on the joint distribution of premiums and risk types of consumers selected into firm j. To simplify notation, let p_j and θ_j be the premiums and risk types of consumers selected into firm j. Let m_j denote the distribution of θ_j, which can be directly estimated from the data.

Lemma 1. *Given m_j, the correlation $corr(p_j, \theta_j)$ is monotonically decreasing in σ_j.*

Proof. Since p_j and θ_j are positively correlated (empirically verifiable),

$$corr(p_j,\theta_j) = \frac{cov(p_j,\theta_j)}{\sqrt{var(\theta_j)var(p_j)}} = \sqrt{\frac{cov^2(p_j,\theta_j)}{var(p_j)var(\theta_j)}} = \sqrt{\frac{var(E(\theta_j|\hat{\theta}_j))}{var(\theta_j)}}. \quad \text{(D.4)}$$

The last equality in Equation (D.4) holds because, by Equation (4.5)

$$var(E(\theta_j|\hat{\theta}_j)) = \frac{var(p_j)}{\beta_j^2} = \frac{var(p_j)}{[var(p_j)/cov(p_j,\theta_j)]^2} = \frac{cov^2(p_j,\theta_j)}{var(p_j)}.$$

Thus, for a fixed type distribution m_j within firm j, it is equivalent to show $var(E(\theta_j|\hat{\theta}_j))$ is monotonically decreasing in σ_j.

We first show that $var(E(\theta_j|\hat{\theta}_j)) = var(E(\theta_j|\hat{\theta}_j^*))$, where $\hat{\theta}_j^* \equiv \hat{\theta}_j + \gamma\beta_j b_j \sigma_j^2$. Let φ

denote the density of $\hat{\theta}_j$ conditional on $D = j$. We have

$$\begin{aligned}
var(E(\theta_j|\hat{\theta}_j)) &= \int (E(\theta_j|\hat{\theta}_j = x))^2 \varphi(x) dx - \mu_j^2 \\
&= \int (E(\theta_j|\hat{\theta}_j^* = x + \gamma\beta_j b_j \sigma_j^2))^2 \varphi(x) dx - \mu_j^2 \\
&= \int (E(\theta_j|\hat{\theta}_j^* = y))^2 \varphi(y - \gamma\beta_j b_j \sigma_j^2) dy - \mu_j^2 \\
&= var(E(\theta_j|\hat{\theta}_j^*)),
\end{aligned}$$

where μ_j is the mean of θ_j, i.e., the average risk type of those self-select into firm j.

Next, we show that $var(E(\theta_j|\hat{\theta}_j^*))$ is monotonically decreasing in σ_j. Note that by Equation (D.3), $\hat{\theta}_j^* \sim \mathcal{N}(\theta, \sigma_j^2)$. Take an independent and normally distributed random variable η. Define

$$\hat{\theta}_j^{*\prime} = \hat{\theta}_j^* + \eta.$$

Since $\hat{\theta}_j^*$ and η are independent and normally distributed, $\hat{\theta}_j^{*\prime} \sim \mathcal{N}(\theta, \sigma_j'^2)$ with a larger variance $\sigma_j'^2 > \sigma_j^2$. By independence, the distribution of $E(\theta_j|\hat{\theta}_j^*)$ is the same as the distribution of $E(\theta_j|\hat{\theta}_j^{*\prime}, \eta)$, so $var(E(\theta_j|\hat{\theta}_j^*)) = var(E(\theta_j|\hat{\theta}_j^{*\prime}, \eta))$. Moreover,

$$var(E(\theta_j|\hat{\theta}_j^{*\prime}, \eta)) > var(E(\theta_j|\hat{\theta}_j^{*\prime})),$$

because on the left-hand side of the inequality we project θ_j onto a larger space. We therefore obtain the desired result that $var(E(\theta_j|\hat{\theta}_j^*)) > var(E(\theta_j|\hat{\theta}_j^{*\prime}))$. □

For a fixed distribution of risk types within a firm, higher variance in the firm's signal distribution leads to a lower correlation between the premiums and risk types. Intuitively, when the signal distribution is very informative, consumers' risk types are better reflected in their premiums, and vice versa.[6] Since the type distribution m_j and $corr(p_j, \theta_j)$ can be easily estimated from the data, the one-to-one mapping between $corr(p_j, \theta_j)$ and σ_j^2 in Lemma 1 uniquely pins down the signal variance.

[6]To see this from another angle, the proof of Lemma 1 shows that $var(E(\theta_j|\hat{\theta}_j))$ decreases with σ_j^2. When σ_j^2 is small, the signals received by the firm are precise, and therefore the posterior mean of θ is very sensitive to the signals. As a result, the variance of the posterior mean is large. By contrast, when σ_j^2 is large, the signals received by the firm are not informative, and therefore the posterior mean is similar across different signals. As a result, the variance of $E(\theta_j|\hat{\theta}_j)$ is small.

E Model Fit

Using the estimates reported in Tables 1 and 2 of the main text, we simulate the premiums offered by each firm and consumers' plan choices given the simulated price menu. Table E.1 presents the means and standard deviations of risk and premium within each firm, using both real and simulated data. Overall, our model matches the key moments of the data reasonably well. Our model-generated price distributions are very close to what we observe in the real dataset. As for the risk sorting pattern, we are able to match the mean and variance of consumers' risks for the majority of firms, and the ranking of average risk levels across firms is largely preserved.[7]

We also evaluate the out-of-sample fit of our model. Specifically, we randomly select 80% of the observations to estimate the model parameters, then simulate premiums and consumer choices for the remaining 20% of the sample. Table E.2 compares observed moments in the testing dataset with those simulated using our estimates. Again, we find that the model replicates key out-of-sample patterns well.

Our estimation relies on the assumption that the premium is a monotonically increasing function of the firm's signal. We show that this assumption is self-consistent by validating the monotone relationship between the risk rating $E_j(\theta|\hat{\theta}_j, D = j)$—which is computed post-estimation—and the signal $\hat{\theta}_j$. In Figure E.1, the risk rating, and therefore the premium, indeed monotonically increases with the signal of Firm 1, for both young and senior drivers. Similar patterns are observed for all other firms.

Finally, we provide external validation of our estimation results. Since firms' risk-rating technologies are proprietary and generally unobservable, we turn to indirect measures to assess the plausibility of our findings. To proxy for the sophistication of a firm's actuarial team, we use LinkedIn data to count the number of employees at major insurers who list expertise in machine learning, data science, or artificial intelligence. The idea is that a larger number of such engineers indicates more advanced pricing capabilities. In Figure E.2a, we plot the firms' information precision rankings against the rankings based on the number of these engineers and find a moderately positive correlation.

Second, to proxy for service quality, we hand-collect data on the number of service centers each major company operates in the Rome metropolitan area, using information from their websites. We compare this measure to our estimates of firms' unobserved product quality (ξ) and find a positive rank correlation, as shown in Figure E.2b.

Third, we validate our marginal cost estimates using firms' financial statements. We

[7]The simulated market shares for low- and high-risk consumers also closely match the patterns observed in the data.

Table E.1: Model fit: Comparing data moments with simulated results using model estimates

(A): Data Moments				
Firm ID	Risk (€)		Premium (€)	
	Mean	Std. Dev.	Mean	Std. Dev.
1	499.86	452.28	519.41	209.20
2	492.41	482.88	556.93	221.56
3	443.83	440.64	568.17	215.58
4	512.60	466.64	427.57	199.30
5	424.22	457.41	506.42	201.00
6	517.59	491.72	448.34	173.35
7	498.45	491.92	526.00	208.25
8	659.83	504.36	576.96	227.48
9	576.66	585.70	551.46	224.70
10	483.60	505.35	523.21	211.78
11	493.60	488.22	430.72	197.85

(B): Simulated Moments Using Model Estimates				
Firm ID	Risk (€)		Premium (€)	
	Mean	Std. Dev.	Mean	Std. Dev.
1	509.84	485.84	509.01	219.60
2	500.17	488.72	539.52	235.64
3	460.28	460.80	550.99	230.75
4	515.58	483.77	423.76	218.02
5	452.92	485.62	508.31	222.33
6	510.80	482.12	434.19	182.62
7	521.76	499.82	514.10	217.81
8	605.84	500.19	536.30	242.47
9	514.71	456.28	513.21	233.13
10	488.03	480.98	507.44	218.85
11	489.59	476.88	433.29	202.53

Table E.2: Out-of-Sample Model fit: Comparing data moments with simulated results using model estimates

(A): Data Moments

Firm ID	Risk (€)		Premium (€)	
	Mean	Std. Dev.	Mean	Std. Dev.
1	491.54	433.31	518.21	209.08
2	469.38	473.34	551.70	227.42
3	439.84	437.73	569.71	215.55
4	519.26	518.50	422.11	193.13
5	413.86	463.96	509.75	208.92
6	522.25	513.97	453.88	176.49
7	508.08	523.94	523.45	205.30
8	657.99	528.12	560.24	225.17
9	520.82	537.65	549.24	218.20
10	481.89	542.93	520.13	215.83
11	502.96	497.22	431.61	196.74

(B): Simulated Moments Using Model Estimates

Firm ID	Risk (€)		Premium (€)	
	Mean	Std. Dev.	Mean	Std. Dev.
1	519.76	483.72	517.13	222.65
2	500.00	498.94	545.04	238.90
3	452.54	453.30	544.16	228.07
4	517.04	485.32	426.80	222.97
5	454.59	498.71	507.76	224.42
6	497.24	473.24	432.61	186.95
7	501.33	509.23	516.32	217.94
8	617.42	512.31	539.57	247.00
9	499.14	434.08	506.36	227.00
10	449.04	451.49	518.96	228.27
11	495.87	497.66	439.21	205.51

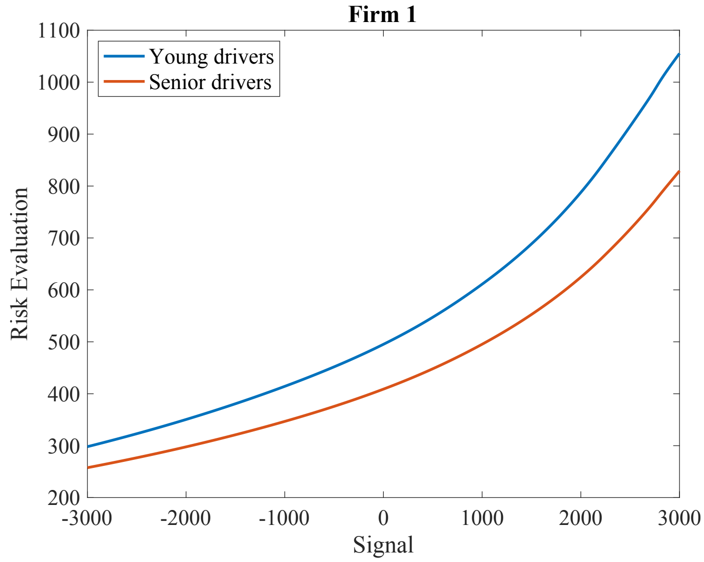


Figure E.1: Risk rating versus signal for Firm 1: The y-axis represents the risk rating, and the x-axis denotes the signal. The blue and red curves represent young and senior drivers, respectively.

collect data from the balance sheets of seven major insurers on their reported expenditures related to customer service and claim liquidation. These cost components closely match our estimates of marginal cost and claim processing efficiency. Our average estimated marginal cost (62 euros) for these companies aligns closely with the reported average (68 euros). Furthermore, the firm-level rankings based on balance sheet data are highly correlated with the rankings derived from our estimated marginal costs and claim efficiency parameters, as shown in Figures E.2c and E.2d.

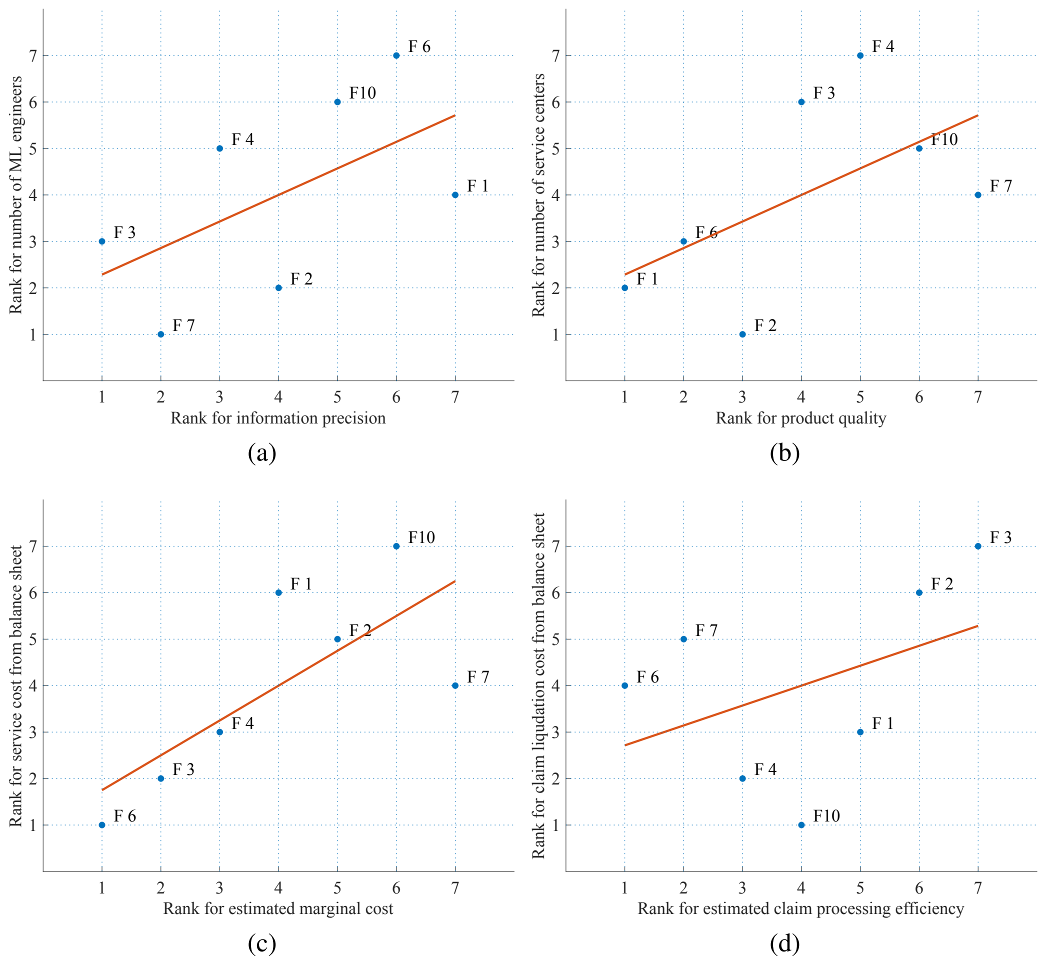


Figure E.2: Rank correlations between model estimates and external measures for seven major insurers. Panel (a): Firms ranked by information precision (x-axis) and number of ML engineers (y-axis), both from highest to lowest. Panel (b): Firms ranked by average product quality (x-axis) and number of service centers (y-axis), both from highest to lowest. Panel (c): Firms are ranked on the x-axis by estimated marginal cost and on the y-axis by service-related expenses (as a percentage of total cost, based on balance sheet data), with both axes ordered from lowest to highest. Panel (d): Firms are ranked on the x-axis by estimated claim processing efficiency and on the y-axis by claim liquidation costs (measured as a percentage of total cost from balance sheet data), with both axes ordered from lowest to highest. The red solid line in each figure represents a linear fit between the two rankings. Firm IDs are displayed next to each dot; firms with incomplete data are excluded from the analysis.

F Solving the Equilibrium for Counterfacual Analysis

To evaluate counterfactual policies, we need to solve the market equilibrium for any given set of model parameters. We propose an iterative procedure to solve for all firms' pricing coefficients $(\boldsymbol{\alpha}, \boldsymbol{\beta})$, the equilibrium expectation $E(\theta|\hat{\theta}_j, D = j)$, and the selection probabilities $Pr(D = j|\hat{\theta}_j, \theta)$ for all j. The iterative algorithm for solving the equilibrium works as follows:

1. In the outer loop, we solve for firms' pricing coefficients $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. Denote the pricing coefficients at the r-th iteration by $(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)$.

2. Given the model primitives $(\gamma, \boldsymbol{\sigma}, \boldsymbol{\xi}, \boldsymbol{c}, \boldsymbol{k})$ and pricing coefficients $(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)$, we solve the r-th equilibrium sorting pattern in an inner loop:

   (1) Set the initial value $E^{r,0}(\theta|\hat{\theta}_j, D = j) = E^{r-1}(\theta|\hat{\theta}_j, D = j)$.

   (2) Compute the premium offered by each firm using the following equation:

   $$p_j^{r,0}(\hat{\theta}_j) = \alpha_j^r + \beta_j^r E^{r,0}(\theta|\hat{\theta}_j, D = j).$$

   (3) Compute the choice probabilities:

   $$Pr^{r,0}(D = j|\hat{\theta}_j, \theta) = \int_{\hat{\boldsymbol{\theta}}_{-j}} \frac{\exp(-\gamma(\theta)p_j^{r,0}(\hat{\theta}_j) + \xi_j(\theta))}{\sum_{j'=1}^{J} \exp(-\gamma(\theta)p_{j'}^{r,0}(\hat{\theta}_{j'}) + \xi_{j'}(\theta))} f(\hat{\boldsymbol{\theta}}_{-j}|\theta; \boldsymbol{\sigma}_{-j}) d\hat{\boldsymbol{\theta}}_{-j}.$$

   (4) Update the expectation by

   $$E^{r,1}(\theta|\hat{\theta}_j, D = j) = \frac{\int_\theta \theta Pr^{r,0}(D = j|\hat{\theta}_j, \theta)\phi(\hat{\theta}_j; \theta, \sigma_j) f_0(\theta) d\theta}{\int_\theta Pr^{r,0}(D = j|\hat{\theta}_j, \theta)\phi(\hat{\theta}_j; \theta, \sigma_j) f_0(\theta) d\theta}.$$

   (5) Iterate this process until the expectation converges. Denote the limits by $E^r(\theta|\hat{\theta}_j, D = j)$ and $Pr^r(D = j|\hat{\theta}_j, \theta)$. These represent the r-th equilibrium sorting pattern.

3. Having obtained $E^r(\theta|\hat{\theta}_j, D = j)$ and $Pr^r(D = j|\hat{\theta}_j, \theta)$ from the inner loop, we now compute the first-order derivatives of the profit function with respect to pricing

coefficients (α_j^r, β_j^r) for all firms. Specifically,

$$\begin{aligned}
\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \alpha_j^r} &= \int_\theta \int_{\hat{\boldsymbol{\theta}}} Pr^r(D=j|\hat{\boldsymbol{\theta}}, \theta) f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta \\
&+ \quad \int_\theta \int_{\hat{\boldsymbol{\theta}}} (\alpha_j^r + \beta_j^r \theta + c_j - k_j \theta) \frac{\partial Pr^r(D=j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \alpha_j^r} f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta, \\
\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \beta_j^r} &= \int_\theta \int_{\hat{\boldsymbol{\theta}}} \theta Pr^r(D=j|\hat{\boldsymbol{\theta}}, \theta) f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta \\
&+ \quad \int_\theta \int_{\hat{\boldsymbol{\theta}}} (\alpha_j^r + \beta_j^r \theta + c_j - k_j \theta) \frac{\partial Pr^r(D=j|\hat{\boldsymbol{\theta}}, \theta)}{\partial \beta_j^r} f(\hat{\boldsymbol{\theta}}|\theta) f_0(\theta) d\hat{\boldsymbol{\theta}} d\theta.
\end{aligned}$$

4. Update the pricing coefficients as follows:

$$\begin{aligned}
\alpha_j^{r+1} &= \alpha_j^r + \Delta\alpha_j^r \mathbf{1}\left\{\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \alpha_j^r} \geq 0\right\} - \Delta\alpha_j^r \mathbf{1}\left\{\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \alpha_j^r} < 0\right\}, \\
\beta_j^{r+1} &= \beta_j^r + \Delta\beta_j^r \mathbf{1}\left\{\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \beta_j^r} \geq 0\right\} - \Delta\beta_j^r \mathbf{1}\left\{\frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \beta_j^r} < 0\right\},
\end{aligned}$$

where $\Delta\alpha_j^r$ and $\Delta\beta_j^r$ are the r-th incremental changes in the pricing coefficients.

5. Finally, we reduce the size of incremental changes $(\Delta\alpha_j^r, \Delta\beta_j^r)$ if sign switching in the first-order conditions is observed in two consecutive iterations.

$$\begin{aligned}
\Delta\alpha_j^{r+1} &= \begin{cases} \frac{\Delta\alpha_j^r}{2} & \text{if} \quad \frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \alpha_j^r} \frac{\partial \pi_j^{r-1}(\boldsymbol{\alpha}^{r-1}, \boldsymbol{\beta}^{r-1})}{\partial \alpha_j^{r-1}} < 0 \\ \Delta\alpha_j^r & \text{otherwise,} \end{cases} \\
\Delta\beta_j^{r+1} &= \begin{cases} \frac{\Delta\beta_j^r}{2} & \text{if} \quad \frac{\partial \pi_j^r(\boldsymbol{\alpha}^r, \boldsymbol{\beta}^r)}{\partial \beta_j^r} \frac{\partial \pi_j^{r-1}(\boldsymbol{\alpha}^{r-1}, \boldsymbol{\beta}^{r-1})}{\partial \beta_j^{r-1}} < 0 \\ \Delta\beta_j^r & \text{otherwise,} \end{cases}
\end{aligned}$$

6. Iterate this process until $(\Delta\alpha_j^r, \Delta\beta_j^r)$ converge to 0 for all j.

Finally, given that the current solution of the pricing coefficient might be a local maximizer, we check whether it is globally optimal. That is, for each firm j, fixing other firms' pricing strategy at the current solution, we search for the value of (α_j, β_j) that maximizes profit π_j. If the global maximizer differs from the current pricing coefficient, we update firm j's pricing coefficient to that global maximizer and repeat steps 1–6. Otherwise, we have found the Nash equilibrium. For the counterfactual scenarios where firms have equal access to either the aggregated risk score from the centralized risk bureau or the true risk type, we solve the equilibrium using a similar iterative procedure, but can skip Step 2.

G Robustness Check: Limited Product Consideration

Our demand model assumes that consumers consider all available insurance products. In practice, however, consumers may face limited consideration sets due to search frictions or other cognitive costs. As a robustness check of our main results, we consider an alternative scenario in which consumers have limited consideration sets. Specifically, we focus on a subsample of consumers who purchase insurance products from the top four firms in the market (Firms 1, 2, 3, and 6), which together account for approximately 42% of total market share. We then re-estimate the demand parameters and the offered price distributions for these firms.

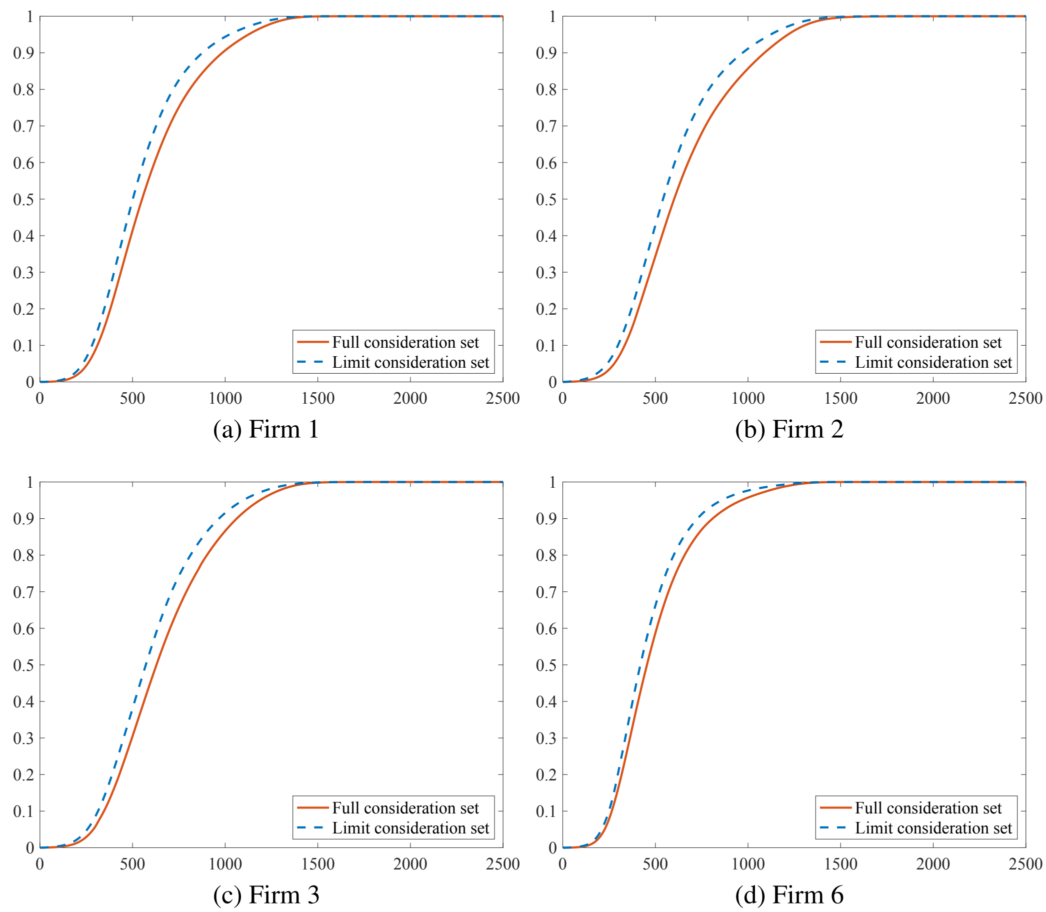
In Table G.1, we compare the demand estimates, including price sensitivity parameters and preferences for unobserved product quality, under full and limited consideration sets. In the estimation, we allow price sensitivity to vary with consumer demographics, such as age and whether the consumer lives in a major city. We also estimate preferences for unobserved product attributes separately for eight demographic groups. The table shows that the demand parameters are broadly similar across the two specifications. However, under the limited consideration set, consumers appear to be slightly less sensitive to price increases.

Another key output from our demand estimation is the distribution of offered prices. Figure G.1 plots the CDF of offered prices for the top four firms under both full and limited consideration assumptions. The distributions again appear quite similar across the two specifications. Taken together, these results suggest that while consumers may not consider all available insurance products in practice, the impact of this limitation on our estimation results is modest.

Table G.1: Comparing demand estimates under full or limited consideration set

(A) Full consideration set								
γ_0	2.11							
Age	-1.21							
Larger urban center	0.45							
	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
ξ_2	-0.54	-0.52	-0.88	-0.76	-0.95	-0.90	-1.41	-1.36
ξ_3	-0.38	-0.62	-0.51	-0.58	-1.49	-1.60	-1.58	-1.64
ξ_6	-0.31	-0.21	-0.36	-0.43	-0.70	-0.56	-0.51	-0.57
(B) Limited consideration set								
γ_0	1.59							
Age	-2.69							
Larger urban center	0.29							
	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
ξ_2	-0.51	-0.61	-0.87	-0.82	-0.92	-0.98	-1.43	-1.49
ξ_3	-0.33	-0.69	-0.47	-0.68	-1.51	-1.74	-1.63	-1.75
ξ_6	-0.19	-0.17	-0.23	-0.29	-0.54	-0.51	-0.40	-0.41
High risk	N	Y	N	Y	N	Y	N	Y
Larger urban center	N	N	Y	Y	N	N	Y	Y
Later periods	N	N	N	N	Y	Y	Y	Y

Note: Panel (A) presents the demand estimation results under the assumption that consumers consider products from all firms. These results are part of the main estimates reported in Table 1, with the exception that we omit the estimates of unobserved product heterogeneity for the remaining firms. Panel (B) shows the results under the assumption that consumers consider only products from the top four insurers. In the estimation, premiums are represented in thousands of euros. The unobserved product heterogeneity for Firm 1 is normalized to zero across all groups. Demographic characteristics for each group are summarized in the bottom panel.



(a) Firm 1 (b) Firm 2 (c) Firm 3 (d) Firm 6

Figure G.1: CDF of offered prices for the top four firms. The red solid lines correspond to the case where consumers consider products from all firms, while the blue dashed lines correspond to the case where consumers consider only the top four firms. The CDFs are averaged over consumer characteristics and risk levels.

H Counterfactuals: Value of Information

Another interesting counterfactual experiment is to evaluate market outcomes when one firm's information technology is improved, thereby assessing the value of information. A key strength of our model lies in its ability to evaluate the *equilibrium effects* when certain (or all) firms improve their information technology. This equilibrium channel operates in addition to the direct effect, where better information allows a firm to improve its risk assessment and pricing. To disentangle these two channels, we consider two additional counterfactual exercises in this section:

- In the first exercise, we improve Firm 1's information precision to match the best in the market, while holding other firms' pricing strategies fixed. This off-equilibrium scenario isolates the direct effect of better information on Firm 1's pricing and performance.
- In the second exercise, we again improve Firm 1's information precision, but now allow all other firms to adjust their pricing strategies in response. This setup captures both the direct effect and the general equilibrium effect through firm interactions in the market.

Table H.1: Counterfactual Results: Off-Equilibrium vs. Equilibrium Outcomes Following an Improvement in Firm 1's Information Precision

	Baseline	Off Equilibrium	Equilibrium Response
Average CS (€)	-542.83	-535.07	-539.14
		(+1.43%)	(+0.68%)
Average CS: Low risk (€)	-477.24	-466.09	-468.54
		(+2.34%)	(+1.82%)
Average CS: High risk (€)	-608.42	-604.04	-609.74
		(+0.72%)	(-0.22%)
Average premium (€)	461.25	454.39	461.80
		(-1.49%)	(+0.12%)
Average profit (€)	849.58	842.16	850.59
		(-0.87%)	(+0.12%)
HHI	2297.26	2286.63	2306.55
		(-0.46%)	(+0.40%)
Average cost (€)	882.39	882.71	881.08
		(+0.04%)	(-0.15%)

Table H.1 summarizes the off-equilibrium and equilibrium market outcomes following an improvement in Firm 1's information precision. The results show that enhancing one firm's information technology leads to only modest changes in overall market outcomes. To better understand firm-level implications, Table H.2 reports the percentage change in total profit for each firm under the two counterfactual scenarios.

We highlight several key findings from Table H.2. First, improving information precision for a single firm significantly increases that firm's profit, underscoring the value of better information. At the same time, nearly all competing firms experience profit losses. However, when competing firms adjust their pricing in response, their profit losses are mitigated, illustrating the role of strategic responses. Interestingly, Firm 1's profit increases even more under the equilibrium scenario. This may occur because rivals, anticipating Firm 1's improved targeting of low-risk consumers, shift their pricing strategies away from these segments, thereby reducing direct competition.

Table H.2: Percentage Changes in Profit: Off-Equilibrium vs. Equilibrium Outcomes Following an Improvement in Firm 1's Information Precision

Firm ID	Off Equilibrium	Equilibrium Response
1	5.85	7.08
2	-4.16	-2.92
3	-2.60	-2.01
4	-1.67	-0.52
5	-0.93	-0.47
6	-1.86	-0.59
7	-5.90	-4.55
8	-3.46	-4.67
9	3.27	5.39
10	-4.21	-3.01
11	-1.61	-0.59

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